LANDFILL BASE GRADES OPTIMIZATION – A MATHEMATICAL MODEL FOR SENSITIVITY ANALYSIS – CASE I

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Landfill designers deal with many technical parameters for the design of various components of landfills, including foundation settlement, slope stability, settlement of waste, leachate collection system hydraulics, maximum leachate head above the liner, and airspace, to name a few. This article discusses the disposal airspace within the permitted boundaries of a landfill. Airspace is the space within the permitted boundary of the landfill lined areas (if liner required), base grades of disposal cells, and final grades of the landfill, excluding the volume of certain soil layers included in the perimeter berms, bottom lining system, and final cover system. Since the disposal airspace of a landfill is considered the actual asset of the landfill owner, which gets consumed with every cubic yard of waste disposed in the landfill, it would make sense that the designer would try to optimize its design to achieve the largest airspace for the landfill.

During the course of the design of a new landfill or an expansion of an existing landfill, the designer selects certain parameters, through various means, for the design of the landfill boundary, base grades, and final grades. These parameters are normally discussed with the landfill owner or its representative during various coordination meetings. A few of these parameters that are normally left to the engineer to select are the leachate collection pipe slope (pipe slope) and base slope, where they are used to develop grades at the bottom of the landfill. Traditionally, a herringbone design has been used by designers to establish necessary slopes for the gravity flow of leachate toward a collection point (sump) at the lowest point of a disposal cell. Procedures followed for the selection of the pipe and base slopes may vary from one engineer to another. Occasionally, minimum allowable values are required for the pipe slope and base slope in accordance with applicable regulations, which adds another level of complication in the process of selecting the design parameters.

Normally, the landfill owner does not get involved in the details of how the designer selects the pipe and base slopes unless the procedure is described in a design approach report prepared by the designer. Even at that point, due to complicated design aspects of a landfill, the owner may not closely examine the work of the designer to understand impacts of the selected slopes on the construction cost and the value of the airspace created by the designer. Considering that the pipe slope and the base slope have significant impacts on the construction cost of the cell foundation and the airspace, it is often surprising that landfill owners do not get more involved in the final selection of these slopes, or ask for a sensitivity analysis by the design engineer.

Part of the reason that sensitivity analysis is not carried out in many cases is the cost of performing the analysis. Sensitivity analysis for selecting the pipe and base slopes begins with developing various scenarios of landfill base grades, followed by performing volume calculations for various combinations of the scenarios with different slopes. The sensitivity analysis evaluates the impact of the pipe and base slopes on the airspace and construction material quantities. The task of developing several scenarios requires a significant effort and may involve many hours of work for each scenario; as a result, the cost of the design project increases significantly, especially if the design includes many disposal cells.

The formulation discussed in this article can potentially cut back on the cost of a sensitivity analysis in a dramatic way because the formulation provides the means to calculate the volume change (i.e., airspace and construction material quantities) among each of two scenarios simply by plugging in a few numbers in an analytical formula. The remainder of this article provides a description of the model, mathematical formulation, and two numerical examples illustrating the ease of calculations in using the formula.

MODEL

The model used for this article is a rectangular cell with grades in accordance with the herringbone concept.

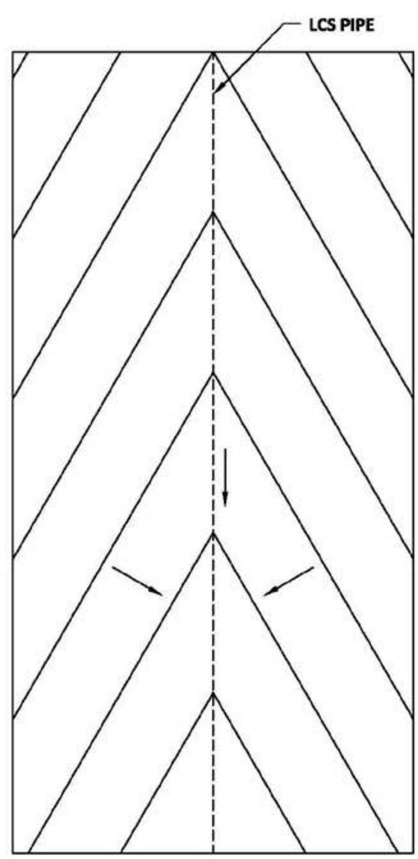

FIGURE 1. BASE GRADE WITH SYMMETRICAL LAYOUT

Figure 1 shows a typical rectangular base of a disposal cell without showing berms on the four sides of the base. The location of the leachate collection pipe is at the trough of the cell base area. To simplify the model, it was assumed that the pipe is not placed inside a trench. The base has two distinct portions, one on either side of the leachate collection pipe. In each portion, two distinct slopes define the grades, namely the pipe slope and the base slope. The basic assumption for this model is that the pipe slope and the base slope remain fixed throughout the entire area of each portion of the cell, but the base slope may vary from one portion to another portion of the cell with the pipe slope being the common element between the two portions (Figure 2).

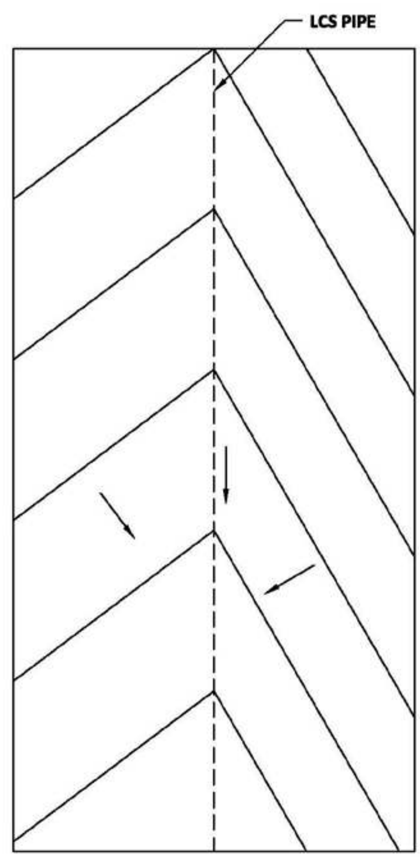

FIGURE 2. BASE GRADE WITH UNSYMMETRICAL LAYOUT

The formulation discussed below is for one portion of the base only. Based on the model described above, the lengths of the cell portions are similar, and the pipe slope remains unchanged from one portion to the other. If the two portions are similar in the base slope and width, the result of the calculations can be multiplied by two to get the result for the entire cell. Alternatively, if the two portions have different base slopes and/or widths, each portion must be analyzed separately and the results must be added to get the final result for the entire cell.

The model considers two sets of grades (surfaces) for analysis: i) the original surface designed or suggested by the engineer for the cell prior to the sensitivity analysis, and ii) the trial surface by a slight variation in the pipe slope and/or the base slope for the sensitivity analysis. The formulation discussed below calculates the volume between the original surface and the trial surface without the need to actually generate a grading plan for the trial surface by graphics software (such as AutoCAD) and performing three dimensional volume calculations between the two surfaces.

FORMULATION

The volume between the original surface and the trial surface may be calculated using differential calculus. An elemental volume presented in Figure 3 is used to define the mathematical relationship between the x, y, and z coordinates of the cell geometry.

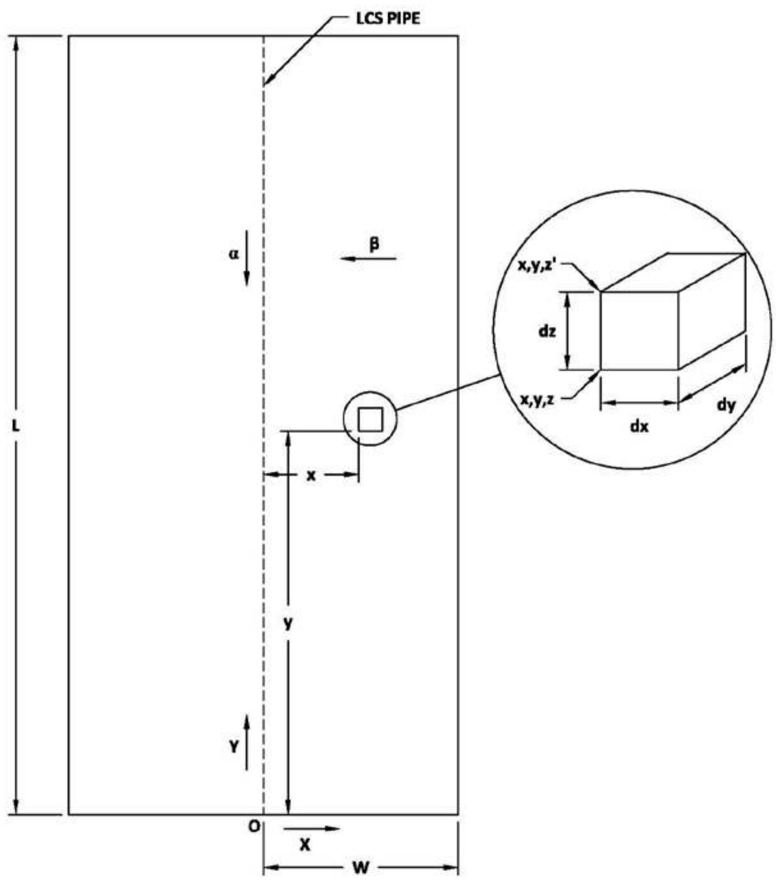

FIGURE 3. ELEMENTAL VOLUME

The elemental volume presented in Figure 3 and the mathematics presented below assumes that the trial surface would be located below the original surface; in other words, the elemental volume would be extending from the original surface down to the trial surface. This arrangement will result in a positive value. The positive value means less construction material will be needed for the base construction and the airspace will increase. The model would still work if the trial surface is located above the original surface, but the result will have a negative value. The negative value means more construction material will be needed for the base construction and the airspace will decrease.

The point of origin is assumed to be at the lowest point of the cell. The x axis extends along the width of the cell and the y axis extends along the length of the cell or the leachate collection pipe. The closest point of the elemental volume on the original surface to the origin of coordinates is located at coordinates x, y, and z, and the closest point of the elemental volume on the trial surface to the origin of coordinates is located at coordinates x, y, and z'. The pipe slope and the base slope for the original surface are α and β, respectively, and the pipe slope and the base slope for the trial surface are α' and β', respectively.

The original surface may be mathematically defined as:

$$z = x \tan\beta + y \tan\alpha$$

The trial surface may be mathematically defined as:

$$z = x \tan\beta' + y \tan\alpha'$$

The volume of the elemental volume, which has infinitesimal dimensions as dx, dy, and dz may be defined in:

$$dv = dx\, dy\, dz$$

Where:

dv represents the volume of the elemental volume.

Using integral calculus, the elemental volume can be expanded over the x, y, and z dimensions to calculate the volume difference between the original and trial surfaces, as mathematically defined below:

$$\int dv = \int dx \int dy \int dz$$

Boundaries for the extension of the above integrals will be from the point of origin (zero point on the coordinate system) to the full width of the portion area (W) along the x axis, to the full length of the cell (L) along the y axis, and to the exterior boundaries of the original and trial surfaces (as defined above) for along the z axis, as defined below:

$$\int dv = \int_o^W dx \int_o^L dy \int_{TS}^{OS} dz$$

where:

OS is the original surface defined by $x \tan\beta + y \tan\alpha$, and

TS is the trial surface defined by $x \tan\beta' + y \tan\alpha'$

Expanding the above equation, using Δv as the integral of the volume difference between the original and trial surfaces:

$$\Delta v = \int_o^W dx \int_o^L dy\, z/_{TS}^{OS}$$

$$\Delta v = \int_o^W dx \int_o^L dy\, [(x \tan\beta + y \tan\alpha) - (x \tan\beta' + y \tan\alpha')]$$

$$\Delta v = \int_o^W dx \int_o^L [x(\tan\beta - \tan\beta') + y(\tan\alpha - \tan\alpha')]dy$$

$$\Delta v = \int_o^W dx \left[\int_o^L x\,(\tan\beta - \tan\beta')dy + \int_o^L y\,(\tan\alpha - \tan\alpha')dy\right]$$

$$\Delta v = \int_o^W (\tan\beta - \tan\beta')x\, dy\, y/_o^L + \int_o^W (\tan\alpha - \tan\alpha')\, dx\, \tfrac{1}{2}\, y^2/_o^L$$

$$\Delta v = L(\tan\beta - \tan\beta') \int_o^W x\, dx + \tfrac{1}{2}\, L^2(\tan\alpha - \tan\alpha') \int_o^W dx$$

$$\Delta v = \tfrac{1}{2}\, L\,(\tan\beta - \tan\beta')x^2/_o^W + \tfrac{1}{2}\, L^2(\tan\alpha - \tan\alpha')x/_o^W$$

$$\Delta v = \tfrac{1}{2}\, LW^2\,(\tan\beta - \tan\beta') + \tfrac{1}{2}\, L^2W(\tan\alpha - \tan\alpha')$$

$$\Delta v = \tfrac{1}{2}\, L\, W[W(\tan\beta - \tan\beta') + L(\tan\alpha - \tan\alpha')]$$

The terms within the bracket represent the differential heights along the cell length and along the half-cell width, respectively, extending from the original surface to the trial surface (or vice versa) at the far ends of the cell length and half-cell width:

$$\Delta l = L(\tan\alpha - \tan\alpha')$$

$$\Delta w = W(\tan\beta - \tan\beta')$$

Where:

Δl = differential height along the cell length, and

Δw = differential height along the cell half-width.

Therefore:

$$\Delta v = \tfrac{1}{2}\, L\, W\, (\Delta l + \Delta w) \qquad (\mathrm{ft}^3) \qquad \text{Equation 1}$$

or:

$$\Delta v = 0.0185\, L\, W\, (\Delta l + \Delta w) \qquad (\text{yd}^3) \qquad \text{Equation 2}$$

Equation 1 or Equation 2 provides the difference in the volume of structural fill from the original surface to the trial surface (less structural fill and more airspace if the trial surface is located below the original surface, and more structural fill or less airspace if the trial surface is located above the original surface) for one-half of the cell total area. In the event excavation must be performed to achieve the finished grades of the original surface (cell base area below existing ground surface), more excavation will be involved if the trial surface is located below the original surface, and less excavation will be involved if reversed.

If the second half of the cell has similar slopes and dimensions, Δv in Equation 1 (or Equation 2) may be multiplied by 2 to obtain the total volume difference for the entire cell area. Otherwise, Equation 1 or Equation 2 may be used to calculate the volume difference for the second half of the cell using the pipe slope, base slope, and half-cell width for the second half of the cell. The sum of the volume differences for the first half and the second half will be the total volume difference for the entire cell.

Equation 1 (or Equation 2) is now a simple tool for the facility owner to perform sensitivity analysis by changing the pipe slope and base slope to any value and determine the financial impact of the changes with respect to the cost of construction and the value of airspace.

Equation 1 may be rearranged in the following form:

$$\Delta v_u = \frac{\Delta v}{LW} = 0.5\, (\Delta l + \Delta w) \qquad (\text{ft}^3/\text{ft}^2) \qquad \text{Equation 3}$$

or,

$$\Delta v_u = 806.6\, (\Delta l + \Delta w) \qquad (\text{yd}^3/\text{acre}) \qquad \text{Equation 4}$$

where:

Δv_u = the unit volume change per unit area of a cell portion.

The above relationship is useful for calculating the average volume change per unit area when the base is changed from an original surface to a trial surface.

The reader should note that the above equations provide an estimate of the volume change. The assumption in the above analysis is that the width and length of the cell (i.e., W and L) would remain unchanged when the design is changed from the original surface to the trial surface. In actuality, W or L slightly change when either of the two slopes (i.e., the pipe slope or the base slope) changes, but the difference is considered small for the purposes of a sensitivity analysis discussed in this article. Additionally, the impact of any changes to the berms on the four sides of the cell as a result of the changes to the pipe slope and the base slope are not included in the above formulation.

EXAMPLE CALCULATIONS

Numerical Example 1:

A rectangular cell with the length of 1000 ft and the width of 400 ft was considered for a sensitivity analysis. The leachate collection pipe is located along the cell centerline. The original slopes for the leachate collection pipe and base are at 1 percent and 2 percent, respectively. How much volume change will be realized if the leachate collection pipe slope and the base slope are changed to 0.5 percent and 1.2 percent, respectively?

Solution:

Cell length = 1000 ft

One-half width = 400 / 2 = 200 ft

Differential height along the cell length = $\Delta l = L(\tan\alpha - \tan\alpha')$

= 1000 (0.01 – 0.005) = 5 ft

Differential height along the cell half-width = $\Delta w = W(\tan\beta - \tan\beta')$

= 200 (0.02 – 0.012) = 1.6 ft

Volume change = $\Delta v = \frac{1}{2}\, L\, W\, (\Delta l + \Delta w)$

= 0.5 x 1000 x 200 x (5 + 1.6)

= 660,000 ft^3 = 24,444 yd^3

Volume change over the entire cell area = 24,444 x 2 = 48,888 yd^3

The above value represents the change in construction material volume and the additional airspace that will be gained when the pipe slope and base slope are changed from 1 percent and 2 percent to 0.5 percent and 1.2 percent, respectively.

The above problem was also solved using the graphical approach by developing the two grading plans discussed above using AutoCAD software. The volume between the two grading plans was calculated using the 3-dimensional volume calculation feature of AutoCAD and the results were similar to the calculated values presented above.

Numerical Example 2:

A rectangular cell with the length of 1000 ft and the width of 400 ft was considered for a sensitivity analysis. The centerline of the cell, where the leachate collection pipe is located, is at 250 ft distance from one side of the cell. The original slope for the leachate collection pipe is at 1 percent. The base slope in the shorter width portion of the cell is at 2 percent, and in the longer width portion of the cell is at 2.5 percent. The sensitivity analysis is intended to keep the leachate collection pipe slope unchanged; however, the base slope in the shorter width portion changes to 1.3 percent and in the longer width portion of the cell changes to 2.1 percent. Calculate the change in volume and unit volume change in each cell portion.

Solution:

For the shorter width portion of the cell:

Δl = 1000 (0.01 – 0.01) = 0.0 ft

Δw = 150 (0.02 – 0.013) = 1.05 ft

Δv = 0.5 x 1000 x 150 (0.0 + 1.05) = 78,750 ft^3 = 2,917 yd^3

Δv_u= 806.6 x (0.0 + 1.05) = 847 yd^3/acre

For the longer width portion of the cell:

Δl = 1000 (0.01 – 0.01) = 0.0 ft

Δw = 250 (0.025 – 0.021) = 1.0 ft

Δv = 0.5 x 1000 x 250 (0.0 + 1.0) = 125,000 ft^3 = 4,630 yd^3

Δv_u= 806.6 x (0.0 + 1.0) = 806.6 yd^3/acre

The total volume change for the entire cell is the sum of the above two volumes:

Total volume change = 2,917 + 4,630 = 7,547 yd^3

Average unit volume change =

= [847 x (150 x 1000) + 806.6 x (250 x 1000)] / (400 x 1000)

= 822 yd^3/acre

The above problem was also solved using the graphical approach by developing the two grading plans discussed above using AutoCAD software. The volume between the two grading plans was calculated using the 3-dimensional volume calculation feature of AutoCAD and the results were similar to the calculated values presented above.

Assuming that the original existing ground is lower than the design grades and structural fill material must be placed over the existing ground to achieve the design grades, the financial savings in structural fill material as a result of the slight changes to the base slopes are:

Assuming \$15/yd^3 for the cost of in-place structural fill (including purchase, hauling, spreading, compaction, CQA, testing, survey, certification):

Savings = 822 x 15 = \$12,330 per acre

Added airspace value as a result of the slight changes to the base slopes is:

Assuming airspace value = \$30 per in-place yd^3

Added airspace value = 822 x 30 = \$24,660 per acre

Total financial benefit = \$12,330 + \$24,660 = \$36,990 per acre

Therefore, no change to the pipe slope and a slight change to the base slope will generate approximately \$37,000 per acre financial benefit to the facility owner. Extending this value over the entire base area would generate a nearly \$340,000 financial benefit for the landfill owner.

APPLICABILITY

The formulation presented above may be applied to any rectangular shaped cell with known cell dimensions, pipe slope, and base slope. If an original design is available and one is keen to find out the differential volume between the original surface and a trial surface with the pipe slope and/or base slope slightly different that the original surface values, Equation 1 or Equation 2 quickly provides an approximate quantity of the differential volume. As discussed above, the user should be sensitive to the order of calculations when Δl and Δw are calculated. As discussed above, if the trial surface slopes are greater than the original surface slopes, the calculated differential height (i.e., along the cell length or along the cell half-width) would be negative. On the contrary, if the trial surface slopes are less than the original surface slopes, the calculated differential height would be positive.

In the event the cell (or the half-cell considered in the evaluation) is not rectangular, the user may consider average values for the cell length and half-cell width to carry on with the analysis. The result of the calculations will be an estimate with a degree of accuracy dependent on the geometry of the irregular cell.

As noted above for the case of dis-similar half cells whereby each half must be calculated separately, the same methodology may be applied when the cell includes more than one leachate collection pipe, in which case, more than two half-cells or partial cells come into the evaluations. Using Equation 1 or Equation 2, the differential volume for each partial cell area should be calculated separately using the pipe and base slopes in that partial cell area.

E-CALCULATIONS

The above formulation can simply be developed in a spreadsheet for a half-cell scenario or a partial cell scenario to expedite calculations, and to eliminate errors that may occur when performing hand calculations. The primary parameters needed for setting up the spreadsheet are:

- Cell length
- Half-cell width or partial-cell width
- Original slope of the pipe
- Original slope of the base
- Trial slope of the pipe
- Trial slope of the base

The spreadsheet may be set up to take information for many partial cells at the same time, and the user would use only as many partial cells that may exist in the cell under consideration.

CONCLUSION

Engineers and landfill owners should both be diligent in maximizing the airspace developed during the design of a new landfill or an expansion to an existing landfill. A sensitivity analysis should be performed by the engineer and closely examined by the landfill owner before important parameters of the design are finalized. Performing a traditional sensitivity analysis could be very expensive to the owner and at times could become prohibitive to the performance of the sensitivity analysis. The analytical formulation presented above provides a simple analytical tool to perform the necessary sensitivity analysis in a short period of time at a low cost.