

# FORMULATION FOR OPTIMIZING LANDFILL BASE SLOPES AND MAXIMIZING AIRSPACE

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**SUMMARY:** This article concentrates on the impact of two parameters that define landfill base grades, namely the leachate collection pipe slope and base slope. In many occasions the designer selects certain pre-determined slope values for the pipe and base of the landfill cells, values that have been used in previous designs or values that have been set by the firm to be used in all designs. This form of selection over rides the necessity of performing a sensitivity analysis to determine impacts of selected slope values on construction cost and available airspace. The task of performing sensitivity analysis can be costly, especially if the design includes many disposal cells. The formulation discussed in the article can potentially cut back on the cost of a sensitivity analysis in a dramatic way. The developed formulation is based on a rectangular cell with grades in accordance with the herringbone concept. The sensitivity analysis begins by considering a second set of slope values for the pipe and base and the formula provides the volume difference between the initial and the second surfaces. This allows the designer to evaluate impact of the pipe and base slopes on the quantity of the structural fill needed for the construction of the cell and the airspace that will be made available for waste disposal.

## 1. INTRODUCTION

Landfill designers deal with many technical parameters for the design of various components of landfills, including foundation settlement, slope stability, settlement of waste, leachate collection system hydraulics, maximum leachate head above the liner, and airspace, to name a few. This article discusses the disposal airspace within the permitted boundaries of a landfill. Airspace is the space within the permitted boundary of the landfill lined areas (if liner required), base grades of disposal cells, and final grades of the landfill, excluding the volume of certain soil layers included in the perimeter berms, bottom lining system, and final cover system. Since the disposal airspace of a landfill is considered the actual asset of the landfill owner, which gets consumed with every cubic yard of waste disposed in the landfill, it would make sense that the designer would try to optimize its design to achieve the largest airspace for the landfill.

During the course of the design of a new landfill or an expansion of an existing landfill, the designer selects certain parameters, through various means, for the design of the landfill boundary, base grades, and final grades. These parameters are normally discussed with the landfill owner or its representative during various coordination meetings. A few of these parameters that are

normally left to the engineer to select are the leachate collection pipe slope (pipe slope) and base slope, where they are used to develop bottom grades of the landfill. Traditionally, a herringbone design has been used by designers to establish necessary slopes for the gravity flow of leachate toward a collection point (sump) at the lowest point of a disposal cell. Procedures followed for the selection of the pipe and base slopes may vary from one engineer to another. Occasionally, minimum allowable values are required for the pipe slope and base slope in accordance with applicable regulations, which adds another level of complication in the process of selecting the design parameters.

Normally, the landfill owner does not get involved in the details of how the designer selects the pipe and base slopes unless the procedure is described in a design approach report prepared by the designer. Even at that point, due to complicated design aspects of a landfill, the owner may not closely examine the work of the designer to understand impacts of the selected slopes on the construction cost and the amount of airspace created by the designer. Considering that the pipe slope and the base slope have significant impacts on the construction cost of the cell foundation and the airspace, it is often surprising that landfill owners do not get more involved in the final selection of these slopes, or ask for a sensitivity analysis by the design engineer.

Part of the reason that sensitivity analysis is not carried out in many cases is the cost of performing the analysis. Sensitivity analysis for selecting the pipe and base slopes begins with developing various scenarios of landfill base grades, followed by performing volume calculations for various combinations of the scenarios with different slopes. The sensitivity analysis evaluates the impact of the pipe and base slopes on the airspace and construction material quantities. The task of developing several scenarios requires a significant effort and may involve many hours of work for each scenario; as a result, the cost of the design project increases significantly, especially if the design includes many disposal cells.

The formulation discussed in this article can potentially cut back on the cost of a sensitivity analysis in a dramatic way because the formulation provides the means to calculate the volume change (i.e., airspace and construction material quantities) among each of two scenarios simply by plugging in a few numbers in an analytical formula. The remainder of this article provides a description of the model, mathematical formulation, and two numerical examples illustrating the ease of calculations in using the formula.

## **2. MODEL**

The model used for this article is a rectangular cell with grades in accordance with the herringbone concept.

Figure 1 shows a typical rectangular base of a disposal cell without showing berms on the four sides of the base. The location of the leachate collection pipe is at the trough of the cell base area. To simplify the model, it was assumed that the pipe is not placed inside a trench. The base has two distinct portions (panels), one on either side of the leachate collection pipe. In each panel, two distinct slopes define the grades, namely the pipe slope and the base slope. The basic assumption for this model is that the pipe slope and the base slope remain fixed throughout the entire area of each panel, but the base slope may vary from one panel to another with the pipe slope being the common element between the two panels (Figure 2).

The model considers two sets of grades (surfaces) for analysis: i) the original surface designed or suggested by the engineer for the cell prior to the sensitivity analysis, and ii) the trial surface by a slight variation in the pipe slope and/or the base slope for the sensitivity analysis. The formulation discussed below calculates the volume between the original surface and the trial surface without the need to actually generate a grading plan for the trial surface by a graphics software (such as AutoCAD) and performing three dimensional volume calculations between the two surfaces.

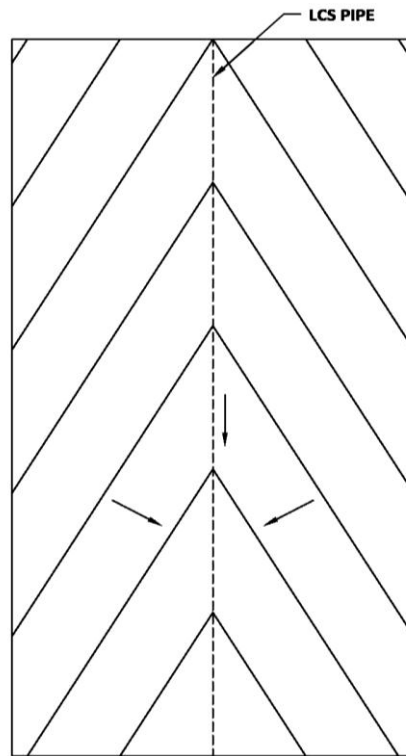


Figure 1. Base grade with symmetrical layout.

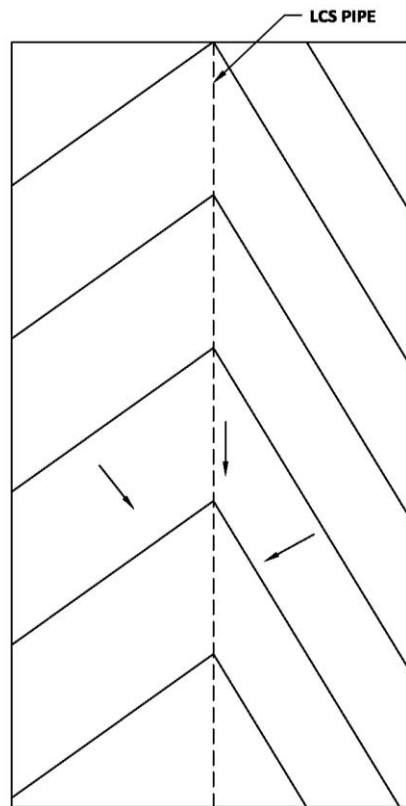


Figure 2. Base grade with unsymmetrical layout

### 3. FORMULATION

The volume between the original surface and the trial surface may be calculated using differential calculus. An elemental volume presented in Figure 3 is used to define the mathematical relationship between the  $x$ ,  $y$ , and  $z$  coordinates of the cell geometry.

The point of origin is assumed to be at the lowest point of the cell. The  $x$  axis extends along the width of the cell and the  $y$  axis extends along the length of the cell or the leachate collection pipe. The closest point of the elemental volume on the original surface to the origin of coordinates is located at coordinates  $x$ ,  $y$ , and  $z$ , and the closest point of the elemental volume on the trial surface to the origin of coordinates is located at coordinates  $x$ ,  $y$ , and  $z'$ . The pipe slope and the base slope for the original surface are  $\alpha$  and  $\beta$ , respectively, and the pipe slope and the base slope for the trial surface are  $\alpha'$  and  $\beta'$ , respectively.

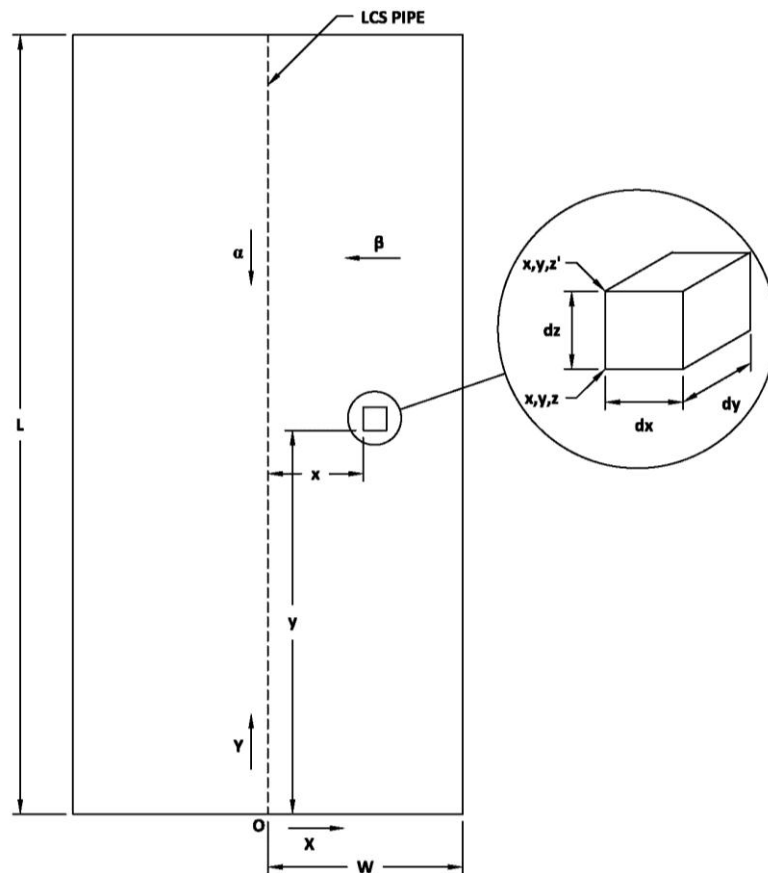


Figure 3. Elemental volume

The original surface may be mathematically defined as:

$$z = x \tan \beta + y \tan \alpha$$

The trial surface may be mathematically defined as:

$$z = x \tan \beta' + y \tan \alpha'$$

The volume of the elemental volume, which has infinitesimal dimensions as  $dx$ ,  $dy$ , and  $dz$  may be defined in:

$$dv = dx dy dz$$

where  $dv$  represents the volume of the elemental volume.

Using integral calculus, the elemental volume can be expanded over the  $x$ ,  $y$ , and  $z$  dimensions to calculate the volume difference between the original and trial surfaces, as mathematically defined below:

$$\int dv = \int dx \int dy \int dz$$

Boundaries for the extension of the above integrals will be from the point of origin (zero point on the coordinate system) to the full width of the portion area ( $W$ ) along the  $x$  axis, to the full length of the cell ( $L$ ) along the  $y$  axis, and to the exterior boundaries of the original and trial surfaces (as defined above) along the  $z$  axis, as defined below:

$$\int dv = \int_0^W dx \int_0^L dy \int_{TS}^{OS} dz$$

Where OS is the original surface defined by  $x \tan \beta + y \tan \alpha$ , and TS is the trial surface defined by  $x \tan \beta' + y \tan \alpha'$

Expanding the above equation, using  $\Delta v$  as the integral of the volume difference between the original and trial surfaces:

$$\begin{aligned} \Delta v &= \int_0^W dx \int_0^L dy z /_{TS}^{OS} \\ \Delta v &= \int_0^W dx \int_0^L dy [(x \tan \beta + y \tan \alpha) - (x \tan \beta' + y \tan \alpha')] \\ \Delta v &= \int_0^W dx \int_0^L [x(\tan \beta - \tan \beta') + y(\tan \alpha - \tan \alpha')] dy \\ \Delta v &= \int_0^W dx \left[ \int_0^L x(\tan \beta - \tan \beta') dy + \int_0^L y(\tan \alpha - \tan \alpha') dy \right] \\ \Delta v &= \int_0^W (\tan \beta - \tan \beta') x dy y /_0^L + \int_0^W (\tan \alpha - \tan \alpha') dx \frac{1}{2} y^2 /_0^L \\ \Delta v &= L(\tan \beta - \tan \beta') \int_0^W x dx + \frac{1}{2} L^2(\tan \alpha - \tan \alpha') \int_0^W dx \\ \Delta v &= \frac{1}{2} L (\tan \beta - \tan \beta') x^2 /_0^W + \frac{1}{2} L^2 (\tan \alpha - \tan \alpha') x /_0^W \\ \Delta v &= \frac{1}{2} LW^2 (\tan \beta - \tan \beta') + \frac{1}{2} L^2 W (\tan \alpha - \tan \alpha') \\ \Delta v &= \frac{1}{2} L W [W(\tan \beta - \tan \beta') + L(\tan \alpha - \tan \alpha')] \end{aligned}$$

The terms within the bracket represent the differential heights along the cell length and along the panel width, respectively, extending from the original surface to the trial surface (or vice versa) at the far ends of the cell length and panel width:

$$m = \tan \alpha - \tan \alpha'$$

$$n = \tan \beta - \tan \beta'$$

Therefore:

$$\Delta v = \frac{1}{2} L W (mL + nW) \quad (m^3) \quad (1)$$

Equation 1 provides the difference in the volume of fill from the original surface to the trial surface for one panel of the cell total area.

If the second panel has similar slopes and dimensions,  $\Delta v$  in Equation 1 may be multiplied by 2 to obtain the total volume difference for the entire cell area. Otherwise, Equation 1 may be used to calculate the volume difference for the second panel using the pipe slope, base slope, and panel width for the second portion of the cell. The sum of the volume differences for the two panels will be the total volume difference for the entire cell.

Equation 1 is now a simple tool to perform sensitivity analysis by changing the pipe slope and base slope to any value and determine the financial impact of the changes with respect to the cost of construction and the value of airspace.

Equation 1 may be rearranged in the following form:

$$\Delta v_u = \frac{\Delta v}{LW} = 0.5 (mL + nW) \quad (\text{m}^3/\text{m}^2) \quad (2)$$

Where  $\Delta v_u$  = the unit volume change per unit area of a panel.

The above relationship is useful for calculating the average volume change per unit area when the base is changed from an original surface to a trial surface.

The reader should note that the above equations provide an estimate of the volume change. The assumption in the above analysis is that the width and length of the panel (i.e.,  $W$  and  $L$ ) would remain unchanged when the design is changed from the original surface to the trial surface. In actuality,  $W$  or  $L$  slightly change when either of the two slopes (i.e., the pipe slope or the base slope) changes, but the difference is considered small for the purposes of a sensitivity analysis discussed in this article. Additionally, the impact of any changes to the berms on the four sides of the cell as a result of the changes to the pipe slope and the base slope are not included in the above formulation.

## 4. EXAMPLES

### 4.1 Numerical Example 1

A rectangular cell with the length of 300 m and the width of 100 m was considered for a sensitivity analysis. The leachate collection pipe is located along the cell centerline. The original slopes for the leachate collection pipe and base are at 1 percent and 2 percent, respectively. How much volume change will be realized if the leachate collection pipe slope and the base slope are changed to 0.5 percent and 1.2 percent, respectively?

*Solution:*

Cell length = 300 m

Panel width =  $100 / 2 = 50$  m

$m = \tan \alpha - \tan \alpha' = 0.01 - 0.005 = 0.005$

$n = \tan \beta - \tan \beta' = 0.02 - 0.012 = 0.008$

Volume change =  $\Delta v = \frac{1}{2} L W (mL + nW)$

$= 0.5 \times 300 \times 50 \times (0.005 \times 300 + 0.008 \times 50) = 14,250 \text{ m}^3$

Volume change over the entire cell area =  $14,250 \times 2 = 28,500 \text{ m}^3$

The above value represents the change in construction material volume and the additional airspace that will be gained when the pipe slope and base slope are changed from 1 percent and 2 percent to 0.5 percent and 1.2 percent, respectively.

The above problem was also solved using the graphical approach by developing the two grading plans discussed above using AutoCAD software. The volume between the two grading plans was calculated using the 3-dimensional volume calculation feature of AutoCAD and the results were similar to the calculated values presented above.

## 4.2 Numerical Example 2

A rectangular cell with the length of 300 m and the width of 100 m was considered for a sensitivity analysis. The centerline of the cell, where the leachate collection pipe is located, is at 70 m distance from one side of the cell. The original slope for the leachate collection pipe is at 1 percent. The base slope in the panel with shorter width is at 2 percent, and in the panel with longer width is at 2.5 percent. The sensitivity analysis is intended to keep the leachate collection pipe slope unchanged; however, the base slope in the panel with shorter width changes to 1.3 percent and in the panel with longer width changes to 2.1 percent. Calculate the change in volume and unit volume change in each panel.

*Solution:*

For the panel with shorter width:

$$m = 0.01 - 0.01 = 0.0$$

$$n = 0.02 - 0.013 = 0.007$$

$$\Delta v = 0.5 \times 300 \times 30 (300 \times 0.0 + 30 \times 0.007) = 945 \text{ m}^3$$

$$\Delta v_u = 945 / (300 \times 30) = 0.105 \text{ m}^3/\text{m}^2$$

For the panel with longer width:

$$m = 0.01 - 0.01 = 0.0$$

$$n = 0.025 - 0.021 = 0.004$$

$$\Delta v = 0.5 \times 300 \times 70 (300 \times 0.0 + 70 \times 0.004) = 2,940 \text{ m}^3$$

$$\Delta v_u = 2,940 / (300 \times 70) = 0.14 \text{ m}^3/\text{m}^2$$

The total volume change for the entire cell is the sum of the above two volumes:

$$\text{Total volume change} = 945 + 2,940 = 3,885 \text{ m}^3$$

Average unit volume change =

$$= [0.105 \times (30 \times 300) + 0.14 \times (70 \times 300)] / (100 \times 300) = 0.13 \text{ m}^3/\text{m}^2$$

The above problem was also solved using the graphical approach by developing the two grading plans discussed above using AutoCAD software. The volume between the two grading plans was calculated using the 3-dimensional volume calculation feature of AutoCAD and the results were similar to the calculated values presented above.

Assuming that the existing ground is lower than the design grades and structural fill material must be placed over the existing ground to achieve the design grades, the financial savings in structural fill material as a result of the slight changes to the base slopes are:

Assuming \$20/m<sup>3</sup> for the cost of in-place structural fill (including purchase, hauling, spreading, compaction, CQA, testing, survey, certification, etc.):

$$\text{Savings} = 0.13 \times 20 = \$2.6 \text{ per m}^3$$

Added airspace value as a result of the slight changes to the base slopes is:

$$\text{Assuming airspace value} = \$50 \text{ per in-place m}^3$$

$$\text{Added airspace value} = 0.13 \times 50 = \$6.5 \text{ per m}^2$$

$$\text{Total financial benefit} = \$2.6 + \$6.5 = \$9.1 \text{ per m}^2$$

Therefore, no change to the pipe slope and a slight change to the base slope will generate approximately \$9.1 per m<sup>2</sup> financial benefit to the facility owner. Extending this value over the entire base area would generate a nearly \$273,000 financial benefit for the landfill owner.

## **5. APPLICABILITY**

The formulation presented above may be applied to any rectangular shaped cell with known cell dimensions, pipe slope, and base slope. If an original design is available and one is keen to find out the differential volume between the original surface and a trial surface with the pipe slope and/or base slope slightly different than the original surface values, Equation 1 quickly provides an approximate quantity of the differential volume. As discussed above, the user should be sensitive to the order of calculations when  $m$  and  $n$  are calculated. As discussed above, if the trial surface slopes are greater than the original surface slopes, the calculated differential angle would be negative. On the contrary, if the trial surface slopes are less than the original surface slopes, the calculated differential angle would be positive.

In the event the cell (or the half-cell considered in the evaluation) is not rectangular, the user may consider average values for the cell length and panel width to carry on with the analysis. The result of the calculations will be an estimate with a degree of accuracy dependent on the geometry of the irregular cell.

As noted above for the case of dis-similar panels whereby each panel must be calculated separately, the same methodology may be applied when the cell includes more than one leachate collection pipe, in which case, more than two panels come into the evaluations. Using Equation 1, the differential volume for each panel should be calculated separately using the pipe and base slopes in that panel.

## **6. E-CALCULATIONS**

The above formulation can simply be developed in a spreadsheet for a panel area to expedite calculations, and to eliminate errors that may occur when performing hand calculations. The primary parameters needed for setting up the spreadsheet are:

- Cell length
- Panel width
- Original slope of the pipe
- Original slope of the base
- Trial slope of the pipe
- Trial slope of the base

The spreadsheet may be set up to take information for many panels at the same time, and the user would use only as many panels that may exist in the cell under consideration.

## **7. CONCLUSION**

Engineers and landfill owners should both be diligent in maximizing the airspace developed during the design of a new landfill or an expansion to an existing landfill. A sensitivity analysis should be performed by the engineer and closely examined by the landfill owner before important parameters of the design are finalized. Performing a traditional sensitivity analysis could be very expensive to the owner, which at times could become prohibitive to the performance of the sensitivity analysis.



The analytical formulation presented above provides a simple analytical tool to perform the necessary sensitivity analysis in a short period of time at a low cost.