Formulation for Sensitivity Analysis to Optimize Design of Pipe and Base Slopes in Landfills with a Double-Segment Herringbone Pattern

By performing a sensitivity analysis, landfill managers can assess financial impacts of the proposed base and pipe slopes.

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Landfill design concepts have significantly matured over the past two decades to the point that optimization of the landfill base grades has become a required exercise during design of landfills. The optimization concept is tied to minimizing the construction cost and maximizing the airspace developed for the landfill. Two parameters that play very important roles in optimizing landfill designs are the leachate collection system pipe (LCS pipe) slope and base slope. The pattern selected for a disposal cell (including the LCS pipe and base slopes) may depend on geology, ground water elevations, geometry of the property and other parameters that fall within the larger design scope of the landfill. However, the general pattern used by designers over the past four decades has been the herringbone pattern.

The herringbone pattern may be fixed (fixed LCS pipe slope and fixed base slope) over the width and length of a panel, where a panel is defined as the area bounded by a berm and a LCS pipe, a ridge in the middle of the disposal cell and a LCS pipe, or any other arrangement that isolates an area from which leachate flows to a LCS pipe. This type of pattern is referred to as a single-segment pattern. Optimization of the LCS pipe and base slopes for a single-segment pattern was discussed in an article published in the August 2014 issue of *Waste Advantage Magazine*. Equations were developed that enable the engineer to change the pipe and/or base slope and calculate an approximate value for the volume between two different patterns (different LCS pipe and/or base slopes). This type of quick calculation provides the ability to perform sensitivity analysis for the LCS pipe and base slope before selecting the final values.

This article deals with a double-segment pattern within a single panel. The double-segment pattern consists of two different herringbone patterns, one following the other in sequence, as shown in **Figure 1, page** 40. The LCS pipe slope in the first segment may be different than the LCS pipe slope in the second segment, and similarly, the base slope in the first segment may be different than the base slope in the second segment. Similar to the model presented in the previous article, the model developed for this article is also a rectangular-shaped area. The formulation developed in this article also allows the base slopes in one panel to be different than the base slopes in another, as shown in **Figure 2, page** 40.

Formulation

The volume between an initial pattern (Pattern 1) and a trial pattern (Pattern 2) may be calculated using differential calculus. Elemental volumes presented in **Figures 3a and 3b, page** 41, are used to define the mathematical relationship between the x, y, and z coordinates of the panel geometry within each segment. For a double-segmented pattern, each segment will be analyzed by its own elemental volume because the distance in the y direction and slopes (pipe and base) in one segment vary independent of the other segment.

The point of origin is assumed to be at the lowest point of the panel. The x axis extends along the width of the panel and the y axis extends along the length of the panel or the leachate collection pipe. Generally, the closest point of the elemental volume in each segment on Pattern 1 to the origin of coordinates is located at coordinates x, y and z, and the closest point of the elemental volume located in each segment on Pattern 2 to the origin of coordinates is located at coordinates x, y and z'. The pipe slope and the base slope in Segment 1 of Pattern 1 are α and β, respectively, and the pipe slope and the base slope in Segment 1 of Pattern 2 are α' and β', respectively. The pipe slope and base slope in Segment 2 of Pattern 1 are δ and y, respectively, and the pipe slope and base slope in Segment 2 of Pattern 2 are δ' and y', respectively.

Pattern 1 within Segment 1 may be mathematically defined as:

$$z = x \tan \beta + y \tan \alpha$$

Pattern 2 within Segment 1 may be mathematically defined as:

$$z' = x \tan \beta' + y \tan \alpha'$$

Pattern 1 within Segment 2 may be mathematically defined as:

$$z = x \tan y + y \tan \delta$$

Pattern 2 within Segment 2 may be mathematically defined as:

$$z' = x \tan y' + y \tan \delta'$$

Volumes of the elemental volumes with infinitesimal dimensions *dx, dy* and *dz* may be defined as:

$$dv_1 = dx\, dy\, dz$$
$$dv_2 = dx\, dy\, dz$$

Where:

dv_1 represents the volume of the elemental volume in Segment 1 and
dv_2 represents the volume of the elemental volume in Segment 2.

Using integral calculus, the elemental volume can be expanded over the *x*, *y* and *z* dimensions within Segments 1 and 2 to calculate the volume difference between Pattern 1 and Pattern 2:

$$\int dv_1 = \int dx \int dy \int dz$$
$$\int dv_2 = \int dx \int dy \int dz$$

Expanding the above integrals over the Segments 1 and 2 boundaries and summing the two volume values will provide the total volume difference between Patterns 1 and 2.

Boundaries for the extension of the above integrals will be from the point of origin (zero point on the coordinate system) to the Segment 1 width (W_1) along the x axis, from the point of origin to the Segment 1 length (L_1) along the z axis, and from Pattern 2 to Pattern 1 within Segment 1 along the z axis:

$$\int dv = \int dv_1 + \int dv_2 = \int_o^{W_1} dx \int_o^{L_1} dy \int_{P_{2-1}}^{P_{1-1}} dz + \int_o^{W_2} dx \int_{L_1}^{L} dy \int_{P_{2-2}}^{P_{1-2}} dz$$

where:

W_1 = width of Segment 1
L_1 = length of Segment 1
W_2 = width of Segment 2
L = length of Pattern 1 or Pattern 2
P_{1-1} is Pattern 1 within Segment 1 defined by $x \tan \beta + y \tan \alpha$, and
P_{2-1} is Pattern 2 within Segment 1 defined by $x \tan \beta' + y \tan \alpha'$
P_{1-2} is Pattern 1 within Segment 2 defined by $x \tan \gamma + y \tan \delta$ and
P_{2-2} is Pattern 2 within Segment 2 defined by $x \tan \gamma' + y \tan \delta'$

Using Δv_1 as the integral of the volume difference between Patterns 1 and 2 within Segment 1 and Δv_2 as the integral of the volume difference between Patterns 1 and 2 within Segment 2, and Δv as the sum of the two volume differences between Patterns 1 and 2 within Segments 1 and 2, the above equation may be expanded and simplified to:

$$\Delta v = \Delta v_1 + \Delta v_2 =$$
$$\tfrac{1}{2}\ L_1\ W_1\ [W_1\ (\tan \beta - \tan \beta') + L_1(\tan \alpha - \tan \alpha')] +$$
$$(\tan \alpha - \tan \alpha')\ L_1\ W_2 (L - L_1) +$$
$$\tfrac{1}{2}\ (\tan \delta - \tan \delta')\ W_2\ (L - L_1)^2 +$$
$$\tfrac{1}{2}\ (\tan \gamma - \tan \gamma')\ W_2^2 (L - L_1) \qquad \text{(Equation 1)}$$

Using the following substitutions:

$m = (\tan \alpha - \tan \alpha')$
$n = (\tan \beta - \tan \beta')$
$p = (\tan \delta - \tan \delta')$
$q = (\tan \gamma - \tan \gamma')$
$L_2 = L - L_1$ = Length of Segment 2

and setting:

$W_1 = W_2 = W$ (rectangular model)

Equation 1 may be simplified to Equation 2 below:

$$\Delta v = \frac{1}{2}\ [m\ W\ L_1^2 + n\ W^2\ L_1 + 2\ m\ W\ L_1\ L_2 + p\ W\ L_2^2 + q\ W^2\ L_2]$$

The Δv value will be in cubic feet and can easily be converted into cubic yards. If the second half of the cell (Panel 2) has similar slopes and dimensions, Δv in Equation 2 may be multiplied by 2 to obtain the total volume difference for the entire cell area. Otherwise, Equation 2 may be used to calculate the volume difference for Panel 2 using the pipe and base slopes in Panel 2. The sum of the volume differences for Panels 1 and 2 will be the total volume difference for the entire cell.

Equation 2 is now a simple tool for engineers or facility owners to perform sensitivity analysis (by changing the pipe and base slopes to any value) and determine the financial impact of the changes with respect to the cost of construction and the value of airspace.

The reader should note that the above equations provide an estimate of the volume change. The assumption in the above analysis is that the widths of Segments 1 and 2 are equal and would remain unchanged when the design is changed from Pattern 1 to Pattern 2. In actuality, W slightly changes when either of the two pipe slopes or base slopes change, but the difference is considered small for the purposes of the sensitivity analysis discussed in this article. Additionally, the impact of any change to the berms on the four sides of the cell as a result of the changes to the pipe slope and base slope are not included in the above formulation.

Figure 1: Panels 1 and 2 with symmetrical patterns.

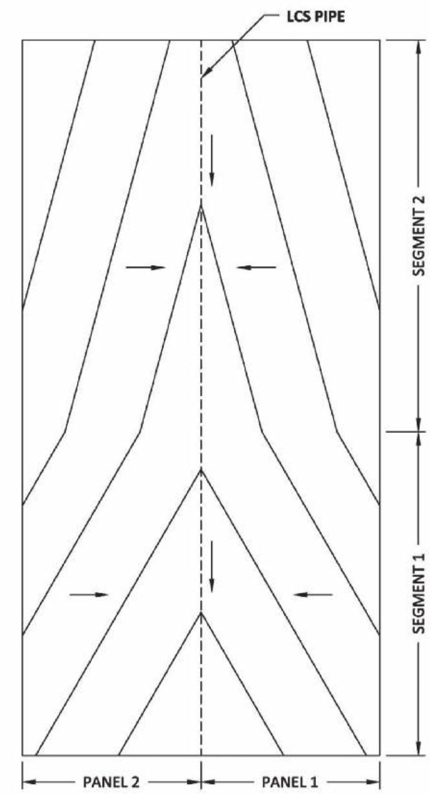
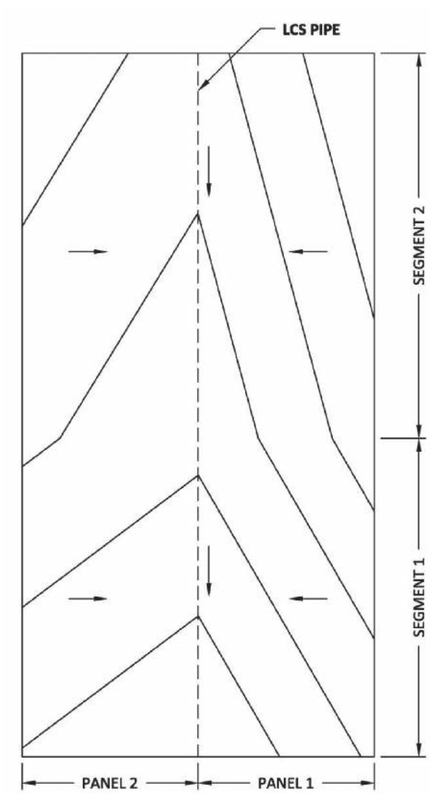


Figure 2: Panels 1 and 2 with unsymmetrical patterns.



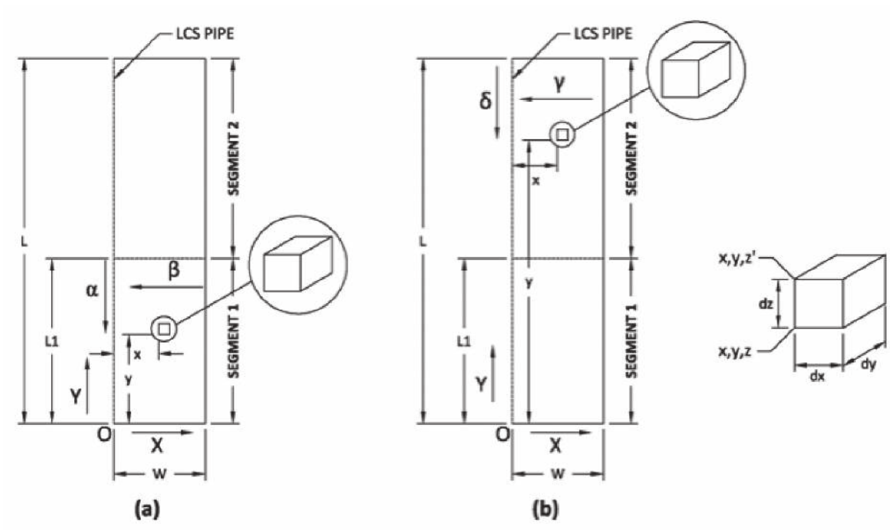


Figure 3: Elemental volume in a segment.
Images courtesy of SCS Engineers.

Conclusion

The above mathematical model may be used by engineers to perform sensitivity analysis for the base and pipe slopes. The sensitivity analysis will reveal the quantity of soil between the selected option and any other option with slightly different base and/or pipe slopes. The sensitivity analysis allows for optimization of the base and pipe slopes to minimize construction cost and maximize airspace.

The above mathematical model may also be used by landfill managers to assess financial impacts of the proposed base and pipe slopes by their engineer. The financial impact may be from the cost of material used for the development of the cell base, and the airspace lost or gained as a result of changes to the base and pipe slopes.

The above mathematical model significantly reduces the cost of performing sensitivity analysis by engineers, and these savings could also be passed through to the landfill owner in the form of lesser technical service fees by the engineer. These savings could potentially be in the range of tens of thousands of dollars. The value of construction cost savings plus the value of additional airspace could potentially be in the range of thousands of dollars to tens of thousands of dollars per acre of the cell area. | WA

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