Formulation for Sensitivity Analysis to Optimize Design of Pipe and Base Slopes in Landfills

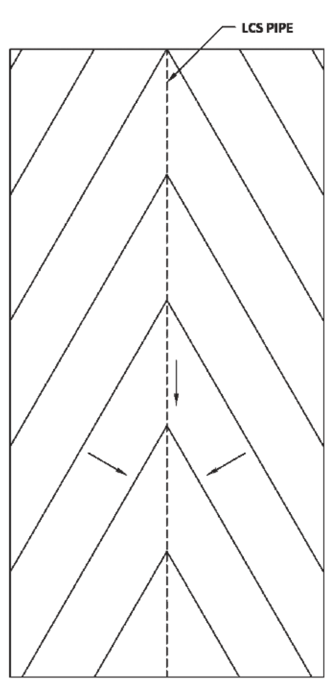
By performing a sensitivity analysis that evaluates the impact of the pipe and base slopes on the airspace and construction material quantities, potential savings could be in the range of several tens of thousands of dollars. The mathematical model may also reduce the cost of construction and increase the value gained from additional airspace for landfill owners.

By Ali Khatami, Ph.D., P.E.

Landfill designers deal with many technical parameters for the design of various components of landfills, including foundation settlement, slope stability, settlement of waste, leachate collection system hydraulics, maximum leachate head above the liner and airspace, to name a few. Airspace is the space within the permitted boundary of the landfill-lined areas, base grades of disposal cells and final grades of the landfill. Since the disposal airspace of a landfill is considered the actual asset of a landfill owner, which gets consumed with every cubic yard of waste disposed in the landfill, it would make sense that the designer would try to optimize its design to achieve the largest airspace for the landfill.

During the course of the design of a new landfill or an expansion of an existing landfill, the designer selects certain parameters, through various means, for the design of the landfill boundary, base grades and final grades. These parameters are normally discussed with the facility owner and decisions are made. Among a few of these parameters that may be left to the engineer to select are the leachate collection pipe slope (pipe slope) and base slope, where they are used to develop grades at the bottom of the landfill. Traditionally, a herringbone design has been used by designers to establish necessary slopes for the gravity flow of leachate toward a collection point (sump) at the lowest point of a disposal cell. Procedures followed for the selection of the pipe and base slopes may vary from one engineer to another. Occasionally, minimum allowable values are required for the pipe and base slopes in accordance with applicable regulations, which adds another level of complication in the process of selecting the design parameters.

Figure 1
Base grade with symmetrical layout.



Images courtesy of SCS Engineers.

Figure 2
Base grade with unsymmetrical layout.

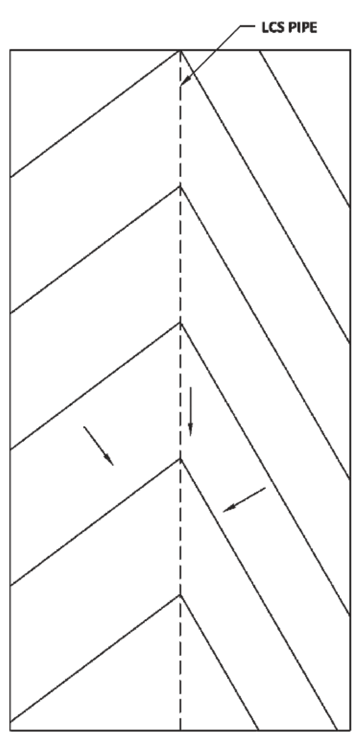
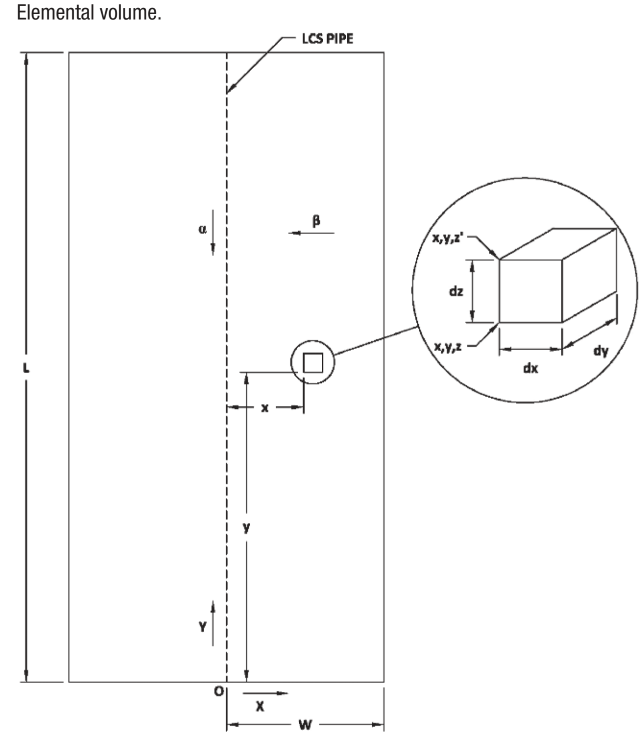


Figure 3
Elemental volume.



In many cases, a sensitivity analysis to optimize the pipe and base slopes is not performed. Part of the reason that sensitivity analysis is not carried out is the cost of performing the analysis. Sensitivity analysis to optimize the pipe and base slopes begins with developing various scenarios of landfill base grades, followed by performing volume calculations for various combinations of the scenarios with different slopes. The sensitivity analysis evaluates the impact of the pipe and base slopes on the airspace and construction material quantities. The task of developing several scenarios requires a significant effort and may involve many hours of work for each scenario; as a result, the cost of the design project increases significantly, especially if the design includes many disposal cells.

The formulation discussed in this article can potentially cut back on the cost of a sensitivity analysis in a dramatic way because the formulation provides the means to calculate the volume change (i.e., airspace and construction material quantities) among each of two scenarios simply by plugging in a few numbers in an analytical formula. The remainder of this article provides a description of the model, mathematical formulation and two numerical examples illustrating the ease of calculations in using the formula.


Thinking out of the box.

Model

The model used for this article is a rectangular cell with grades in accordance with the herringbone concept. **Figure 1** shows a typical rectangular base of a disposal cell without showing berms on the four sides of the base. The location of the leachate collection pipe is at the trough of the cell base area. To simplify the model, it was assumed that the pipe is not placed inside a trench. The base has two distinct portions, one on either side of the leachate collection pipe. In each portion, two distinct slopes define the grades, namely the pipe slope and the base slope. The basic assumption for this model is that the pipe slope and the base slope remain fixed throughout the entire area of each portion of the cell, but the base slope may vary from one portion to another portion of the cell with the pipe slope being the common element between the two portions **(Figure 2)**.

The formulation discussed is for one portion of the base only. Based on the model described above, the lengths of the cell portions are similar, and the pipe slope remains unchanged from one portion to the other. If the two portions are similar in the base slope and width, the result of the calculations can be multiplied by two to get the result for the entire cell. Alternatively, if the two portions have different base slopes and/or widths, each portion must be analyzed separately and the results must be added to get the final result for the entire cell. The model considers two sets of grades (surfaces) for analysis: 1) the original surface designed or suggested by the engineer for the cell prior to the sensitivity analysis, and 2) the trial surface by a slight variation in the pipe slope and/or the base slope for the sensitivity analysis. The formulation discussed calculates the volume between the original surface and the trial surface without the need to actually generate a grading plan for the trial surface by graphics software (such as AutoCAD) and performing three dimensional volume calculations between the two surfaces.

Formulation

The volume between the original surface and the trial surface may be calculated using differential calculus. An elemental volume presented in **Figure 3, page xx** is used to define the mathematical relationship between the x, y, and z coordinates of the cell geometry. The elemental volume presented in Figure 3 and the mathematics presented below assumes that the trial surface would be located below the original surface; in other words, the elemental volume would be extending from the original surface down to the trial surface. This arrangement will result in a positive value. The positive value means less construction material will be needed for the base construction and the airspace will increase. The model would still work if the trial surface is located above the original surface, but the result will have a negative value. The negative value means more construction material will be needed for the base construction and the airspace will decrease.

The point of origin is assumed to be at the lowest point of the cell. The x-axis extends along the width of the cell and the y-axis extends along the length of the cell or the leachate collection pipe. The closest point of the elemental volume on the original surface to the origin of coordinates is located at coordinates x, y, and z, and the closest point of the elemental volume on the trial surface to the origin of coordinates is located at coordinates x, y, and z'. The pipe slope and the base slope for the original surface are α and β, respectively, and the pipe slope and the base slope for the trial surface are α' and β', respectively. The original surface may be mathematically defined as:

$z = x \tan\beta + y \tan\alpha$

The trial surface may be mathematically defined as:

$z = x \tan\beta' + y \tan\alpha'$

The volume of the elemental volume, which has infinitesimal dimensions as $dx\,dy$, and dz may be defined in:

$dv = dx\,dy\,dz$

Where:

dv represents the volume of the elemental volume.

Using integral calculus, the elemental volume can be expanded over the x, y, and z dimensions to calculate the volume difference between the original and trial surfaces, as mathematically defined below:

$$\int dv = \int dx \int dy \int dz$$

Boundaries for the extension of the above integrals will be from the point of origin (zero point on the coordinate system) to the full width of the portion area (W) along the x axis, to the full length of the cell (L) along the y axis, and to the exterior boundaries of the original and trial surfaces (as defined above) for along the z axis, as defined below:

$$\int dv = \int_o^W dx \int_o^L dy \int_{TS}^{OS} dz$$

where:

OS is the original surface defined by $x \tan\beta + y \tan\alpha$, and

TS is the trial surface defined by $x \tan\beta' + y \tan\alpha'$

Using Δv as the integral of the volume difference between the original and trial surfaces, the above equation may be expanded and simplified to:

$$\Delta v = \frac{1}{2} L\,W[W(\tan\beta - \tan\beta') + L(\tan\alpha - \tan\alpha')]$$

The terms within the bracket represent the differential heights along the cell length and along the half-cell width, respectively, extending from the original surface to the trial surface (or vice versa) at the far ends of the cell length and half-cell width:

$$\Delta l = L(\tan\alpha - \tan\alpha')$$
$$\Delta w = W(\tan\beta - \tan\beta')$$

Where:

Δl = differential height along the cell length, and

Δw = differential height along the cell half-width.

Therefore:

$$\Delta v = \frac{1}{2}\, L\, W\, (\Delta l + \Delta w) \qquad (\text{ft}^3) \qquad \text{Equation 1}$$

or:

$$\Delta v = 0.0185\, L\, W\, (\Delta l + \Delta w) \qquad (\text{yd}^3) \qquad \text{Equation 2}$$

Equation 1 or Equation 2 provides the difference in the volume of structural fill from the original surface to the trial surface (less structural fill and more airspace if the trial surface is located below the original surface, and more structural fill or less airspace if the trial surface is located above the original surface) for one-half of the cell total area. In the event excavation must be performed to achieve the finished grades of the original surface (cell base area below existing ground surface), more excavation will be involved if the trial surface is located below the original surface, and less excavation will be involved if reversed.

If the second half of the cell has similar slopes and dimensions, Δv in Equation 1 (or Equation 2) may be multiplied by two to obtain the total volume difference for the entire cell area. Otherwise, Equation 1 or Equation 2 may be used to calculate the volume difference for the second half of the cell using the pipe slope, base slope, and half-cell width for the second half of the cell. The sum of the volume differences for the first half and the second half will be the total volume difference for the entire cell. Equation 1 (or Equation 2) is now a simple tool for the facility owner to perform sensitivity analysis by changing the pipe slope and base slope to any value and determine the financial impact of the changes with respect to the cost of construction and the value of airspace. Equation 1 may be rearranged in the following form:

$$\Delta v_u = \frac{\Delta v}{LW} = 0.5\,(\Delta l + \Delta w) \qquad (\text{ft}^3/\text{ft}^2) \qquad \text{Equation 3}$$

or,

$$\Delta v_u = 806.6\,(\Delta l + \Delta w) \qquad (\text{yd}^3/\text{acre}) \qquad \text{Equation 4}$$

where:

Δv_u = the unit volume change per unit area of a cell portion.

The above relationship is useful for calculating the average volume change per unit area when the base is changed from an original surface to a trial surface. The reader should note that the above equations provide an estimate of the volume change. The assumption in the above analysis is that the width and length of the cell (i.e., W and L) would remain unchanged when the design is changed from the original surface to the trial surface. In actuality, W or L slightly change when either of the two slopes (i.e., the pipe slope or the base slope) changes, but the difference is considered small for the purposes of a sensitivity analysis discussed in this article. Additionally, the impact of any changes to the berms on the four sides of the cell as a result of the changes to the pipe slope and the base slope are not included in the above formulation.

Potential Savings

Engineers may use the mathematical model to minimize construction costs. Performing sensitivity analysis on the selected base and pipe slopes will reveal the quantity of soil between the selected option and other options with a slightly different base and/or pipe slope.

An engineer may also use the mathematical model to assess financial impacts of the proposed base and pipe slopes. The financial impact, as a result of changes to the base and pipe slope, may be from the material used for the development of the cell base area and the airspace lost or gained.

Landfill owners would require fewer technical services by engineers thereby reducing the cost of performing sensitivity analysis. Potential savings could be in the range of several tens of thousands of dollars. The mathematical model may also reduce the cost of construction and increase the value gained from additional airspace for landfill owners. The total cost of construction savings and value added for additional airspace could potentially be in the range of a few thousands of dollars to tens of thousands of dollars per acre of the cell area. | WA

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